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Dislocation damping in the low amplitude linear range due to interacting electron

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A quantitative theory of the attenuation in dislocation motion in the low amplitude linear range due to electron has been solved by the powerful operational method developed by Heaviside (1966). The problem is solved on the idea of vibrating string model of dislocation.

INTRODUCTION

The general quantitative theory of dislocation motion on the basis of vibrating string model was given by Granato & Lücke (1956). They considered a dislocation line with two types of pinning points, namely, nodes of dislocation network on the one hand and impurities and other point imperfections on the other. Neglecting the effect of electrons interacting with dislocation in a metal, either in the normal or superconducting states, Granato & Lücke and Bhattacharya & Ghose (1972) calculated the damping due to dislocation motion caused by periodic stress on the specimen. In reality a strain wave in a metal causes the electron to move in such a way as to establish current neutrality. The result is that the electrons are given a velocity equal to the particle velocity of the acoustic wave The momentum thus imparted to the electrons can be exchanged between surfaces moving with slightly different velocities producing an electron viscosity which damps out further dislocation motion, with a force proportional to the velocity of dislocation.

SOLUTION OF THE PROBLEM

The displacement ξ of the dislocation under the influence of an applied stress as given by the mathematical model for the equation of motion of a pinned-down dislocation loop used by Koehler in the light of Rayleigh's stretched string theory may be expressed as

$$A\frac{\partial^2\xi}{\partial t^2}+B\frac{\partial\xi}{\partial t}-C\frac{\partial^2\xi}{\partial y^2}=a\sigma \tag{1}$$

where A = effective mass per unit length $=\rho a^2$, B = damping constant, C = tension on the dislocation line $\approx Ga^2/2$, a = Burger's vector, σ = applied stress, and ξ is a function of x, y and t and the term on the right is the force per unit length exerted on the dislocation line by external stress.

The boundary conditions for all values of x and t are given by

$$\xi = 0 \quad \text{at} \quad y = 0 \qquad \ldots \quad (2)$$

and

$$\xi = 0 \quad \text{at} \quad y = l. \qquad \ldots \quad (3)$$

Let us assume that the applied stress σ is periodic in time and independent of y and can be represented by

$$\sigma = \sigma_0 \exp(-\alpha x) \exp[i\omega(t-x/v)] \qquad \ldots \quad (4)$$

Simplified operational solution of equation (1) is obtained with the help of equations (2), (3) and (4) and is given by

$$\xi = \mu\left[1-\frac{\cosh \nu y+\cosh \nu(y-l)}{1+\cosh \nu l}\right]\exp(i\omega t) \qquad \ldots \quad (5)$$

where

$$\mu = -Kc_1^2\left[\frac{1}{d^2+\omega^2}+\frac{id}{\omega(d^2+\omega^2)}\right],$$

$$K = \frac{a}{c}\,\sigma_0 \exp\left(-\alpha x-\frac{i\omega x}{v}\right), \quad d = \frac{B}{A}, \quad C_1^2 = \frac{C}{A}$$

and

$$\nu = \frac{i\omega+\frac{1}{2}d}{C_1}$$

This is the most general solution of the displacement of the dislocation loop of length l.

Now when the frequency is low *i.e.*, when $\nu \leqslant 1$ equation (5) after simplification turns out to be,

$$\xi = A'(ly-y^2) \qquad \ldots \quad (6)$$

where

$$A' = \frac{\mu\nu^2}{2}\exp(i\omega t)$$

By substituting (6) in (1) and integrating with respect to y from 0 to l, the constant A' takes the form

$$A' = \frac{\sigma a}{2C\left[1+i\omega\frac{Bl^2}{12C}-\omega^2\frac{Al^2}{12}\right]} \qquad \ldots \quad (7)$$

Remembering that $A = \rho a^2$, $C = \frac{Ga^2}{2}$ and substituting $d_1 = \frac{Bl^2}{6Ga^2}$ and $\omega_r = \frac{1}{l}\left(\frac{6G}{\rho}\right)^{\frac{1}{2}}$ equation (7) turns out to be

$$A' = \frac{\sigma}{Ga\left[1+i\omega d_1-\frac{\omega^2}{\omega_r^2}\right]} \qquad \ldots \quad (8)$$

Since ω_r is much larger than the experimental frequencies employed, the last term in the denominator can be dropped. So (8) takes the form

$$A' = \frac{\sigma}{Ga[1+i\omega d_1]} \qquad \dots \quad (9)$$

Again the average displacement $\bar{\xi}$ of the loop of length l is given by

$$\bar{\xi} = \frac{1}{l}\int_0^l \xi \mathrm{d}y$$

$$= \frac{1}{l}\int_0^l A'(ly-y^2)\mathrm{d}y$$

or

$$\bar{\xi} = \frac{A'l^2}{6} \qquad \dots \quad (10)$$

The effect of N_0 loops per cubic centimeter of average length l_0 on the shear strain ϵ_{sh} is obtained by adding to the elastic strain ϵ_{el} a plastic strain ϵ_{pl} *i.e.*,

$$\epsilon_{sh} = \epsilon_{el}+c_{pl}$$

$$= \epsilon_{el}+N_0 sa \qquad \dots \quad (11)$$

where $\xi_{pl} = N_0 sa$ and s is the area of the loop.

But we know that the area of the loop is

$$s = \bar{\xi}l = \frac{A'l^3}{6} \qquad \dots \quad (12)$$

Denoting shear modulus G' as

$$G' = \frac{\sigma}{\epsilon_{sh}} \qquad \dots \quad (13)$$

and combining equations (11) and (13), we have

$$\frac{1}{G'} = \frac{\epsilon_{sh}}{\sigma}$$

$$= \frac{\epsilon_{el}+(N_0 aA'l^3/6)}{\sigma}$$

$$= \frac{1}{G}\left[1+\frac{N_0 l^3}{6(1+i\omega d_1)}\right]$$

$$= \frac{1}{G}\left[1+\frac{\bar{N}l^2}{6(1+i\omega d_1)}\right] \qquad \dots \quad (14)$$

where $N = N_0 l$ is the total length of dislocation per cubic centimeter lying in the glide plane.

Hence,
$$\frac{G'}{G} = \frac{1}{\left[1+\frac{\bar{N}l^2}{6(1+i\omega d_1)}\right]} \quad \cdots \quad (15)$$

Neglecting the term $[\bar{N}l^2/6(1+i\omega d_1)]$ compared to unity and taking real parts of the ratio G'/G we have from equation (15),

$$\frac{G-G'}{G} = \frac{\bar{N}l^2}{6\left[1+\left(\frac{\omega Bl^2}{6Ga^2}\right)^2\right]} \,. \quad \cdots \quad (16)$$

Calling the imaginary part of the shearing modulus G' as G'' we obtain

$$Q^{-1} = \frac{G''}{G} \quad \frac{\bar{N}l^4\omega B}{36Ga^2\left[1+\left(\frac{\omega Bl^2}{6Ga^2}\right)^2\right]} \quad (17)$$

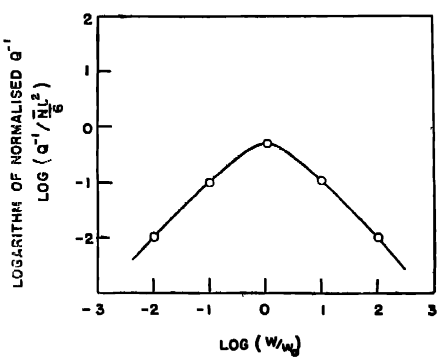
where Q^{-1} is related with the decrement by the relation given by

$$Q^{-1} = \frac{\Delta}{\pi} \,.$$

By putting $\frac{Bl^2}{6Ga^2} = \frac{1}{\omega_0}$, the equation (17) is given by

$$Q^{-1} = \frac{\bar{N}l^2(\omega/\omega_0)}{6[1+(\omega/\omega_0)^2]} \quad (19)$$

The graphical representation of equation (19) *i.e.*, $(Q^{-1}/\bar{N}l^2)$ *vs* (ω/ω_0) is shown in figure 1. Figure 1 agrees with the work of Mason (1966) using single loop length for dislocation damped with electron and phonon viscosity.



LOGARITHM OF NORMALISED FREQUENCY

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